YouTube Lecture Links & Notes:
Unit Test -1 Last Minute Revision Notes for K Scheme: Topic - Integration
YT Link : https://www.youtube.com/watch?v=ux5Lbgl4M6Y

Integration

1) $\int 0 \cdot dx = \text{constant}$

2) $\int 1 \cdot dx = x + c$

3) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

4) $\int k x^n \, dx = k \int x^n \cdot dx$

5) $\int a^x \, dx = \frac{a^x}{\log_e a} + c$

6) $\int e^x \, dx = e^x + c$

7) $\int \frac{1}{x} \, dx = \log x + c$

8) $\int \log x \cdot dx = x \cdot \log x - x + c$

9) $\int x \cdot e^x \, dx = e^x (x - 1) + c$

10) $\int \frac{1}{ax+b} \, dx = \log |ax+b| \cdot \frac{1}{a} + c$

11) $\int \sin x \, dx = -\cos x + c$

12) $\int \cos x \cdot dx = \sin x + c$

13) $\int \tan x \cdot dx = \log |\sec x| + c$

14) $\int \cot x \cdot dx = \log |\sin x| + c$

15) $\int \sec x \, dx = \log |\sec x + \tan x| + c$

16) $\int \operatorname{cosec} x \cdot dx = \log |\operatorname{cosec} x - \cot x| + c$

17) $\int \sec^2 x \cdot dx = \tan x + c$

18) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

19) $\int \sec x \cdot \tan x \, dx = \tan x + c$

20) $\int \operatorname{cosec} x \cdot \cot x = -\cot x + c$

21) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c$

22) $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

23) $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log\left|x + \sqrt{x^2-a^2}\right| + c$

24) $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log\left|x + \sqrt{x^2+a^2}\right| + c$

25) $\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + c$

26) $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \cdot \tan^{-1}\left(\frac{x}{a}\right) + c$

27) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \cdot \log\left|\frac{x-a}{x+a}\right| + c$

28) $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \cdot \log\left|\frac{a+x}{a-x}\right| + c$

(*) 29) $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \cdot \sec^{-1}\left(\frac{x}{a}\right) + c$

30) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \cdot \sin^{-1}\left(\frac{x}{a}\right) + c$

31) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \cdot \log\left|x + \sqrt{x^2-a^2}\right| + c$

32) $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \cdot \log\left|x + \sqrt{x^2+a^2}\right| + c$

(*) 33) $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}x + c$

34) $\int \frac{-1}{1+x^2} dx = \cot^{-1}x + c$

35) $\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1}x + c$

36) $\int \frac{-1}{|x|\sqrt{x^2-1}} dx = \mathrm{cosec}^{-1}x + c$

(37) **Product Rule**

$$\int u \cdot v\, dx = u\int v\, dx - \int \left[\frac{d}{dx}u \cdot \int v\, dx\right] dx$$

L = logarithmic - $\log x$
I = inverse = $\sin^{-1}x$ . . . .
A = algebric = $x / 1 / x^2$ - - - -
T = trignometric = $\sin x, \cos x$ - - - -
E = exponetial = e^x / e^{5x}

\# 38) $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$ //

$f(x)$ = function (denominator)
$f'(x) = \frac{d}{dx} f(x)$

Definite Integration

1) $\text{Area} = \int_a^b y \cdot dx$

2) $\int_a^b f(x)\, dx = \int_a^b f(a+b-x) \cdot dx$

(b: upper limit; a: lower limit)

Q. $I = \int_{-1}^{+1} \frac{x}{1+x^2}\, dx$

$\Rightarrow$

$f(x) = 1 + x^2$

$f'(x) = \frac{d}{dx} f(x)$

$= \frac{d}{dx}(1+x^2)$

$= \frac{d}{dx}1 + \frac{d}{dx}x^2$

$= 0 + 2 \cdot x^{2-1}$

$f'(x) = 2x$

$I = \int_{-1}^{1} \frac{2}{2} \times \frac{x}{1+x^2}\, dx$

$= \int_{-1}^{1} \frac{1}{2} \cdot \frac{2x}{1+x^2}\, dx$

$= \frac{1}{2} \int_{-1}^{1} \frac{2x}{1+x^2}\, dx$

$= \frac{1}{2}\left[\log|1+x^2|\right]_{-1}^{+1}$

$= \frac{1}{2}\left\{\log|1+1^2| - \log|1+(-1)^2|\right\}$

$= \frac{1}{2}\left\{\log 2 - \log 2\right\}$

$= \frac{1}{2}\{0\}$

$I = 0$

$\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + c$

$\frac{d}{dx}\text{constant} = 0$

$\frac{d}{dx} x^n = n \cdot x^{n-1}$

Q. $I = \int_{-1}^{1} \frac{1}{1+x^2} dx$

$\Rightarrow I = \int_{-1}^{1} \frac{1}{1+x^2} dx$

$= \left[\tan^{-1}(x)\right]_{-1}^{+1}$

$= \left\{\tan^{-1}(+1) - \tan^{-1}(-1)\right\}$

$= \left\{\tan^{-1}(1) - \left[-\tan^{-1}(1)\right]\right\}$

$= \left\{\tan^{-1}(1) + \tan^{-1}(1)\right\}$

$= 2\tan^{-1}(1)$

$= 2 \times \frac{\pi}{4}$

$I = \frac{\pi}{2}$ //

$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$

$\tan^{-1}(-x) = -\tan^{-1}(x)$

$\tan^{-1}(1) = 45^\circ$

$45^\circ \Rightarrow 45 \times \frac{\pi}{180} = \frac{\pi}{4}$

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Q. $I = \int_{a=1}^{b=5} \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}}\, dx \quad \text{--- (1)}$

$\Rightarrow$

$x \Rightarrow a+b-x = 1+5-x = 6-x$

$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

$I = \int_{a=1}^{b=5} \frac{\sqrt{9-(6-x)}}{\sqrt{9-(6-x)} + \sqrt{(6-x)+3}}\, dx$

$= \int_1^5 \frac{\sqrt{9-6+x}}{\sqrt{9-6+x} + \sqrt{6-x+3}}\, dx$

$I = \int_1^5 \frac{\sqrt{3+x}}{\sqrt{3+x} + \sqrt{9-x}}\, dx \quad \text{--- (2)}$

$I = \int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}}\, dx \quad \text{--- (1)}$

eq^n (1) + eq^n (2)

$2I = \int_1^5 \frac{\sqrt{3+x}}{\sqrt{3+x} + \sqrt{9-x}}\, dx + \int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}}\, dx$

$2I = \int_1^5 \frac{(\sqrt{3+x} + \sqrt{9-x})}{(\sqrt{3+x} + \sqrt{9-x})}\, dx$

$2I = \int_1^5 1 . dx$

$\int 1 . dx = x + c$

$2I = [x]_1^5$

$2I = [5-1]$

$2I = 4$

$I = \frac{4}{2}$

$\therefore I = 2$

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Q. $I = \int_0^1 \frac{x}{x+1}\,dx$

$$\Rightarrow I = \int_0^1 \frac{(x+1)-1}{(x+1)}\,dx$$

$$= \int_0^1 \left[\frac{(x+1)}{(x+1)} - \frac{1}{(x+1)}\right] dx$$

$$= \int_0^1 \left[1 - \frac{1}{x+1}\right] dx$$

$$= \int_0^1 1.\,dx - \int_0^1 \frac{1}{x+1}\,dx$$

$$= \left[x\right]_0^1 - \left[\log|x+1|\right]_0^1$$

$$= [1-0] - \left[\log|1+1| - \log|0+1|\right]$$

$$= (1) - \left[\log|2| - \log|1|\right]$$

$$\therefore I = 1 - [\log(2) - 0]$$

$$I = 1 - \log 2 \;//$$

$\int 1.\,dx = x + c$

$\int \frac{1}{ax+b}\,dx = \log|ax+b| \cdot \frac{1}{a} + c$

$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

$\log 1 = 0$

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① Integration by Parts

$$\int u \cdot v \, dx$$

② Integration by Partial fraction

$$\frac{1}{(x-\alpha)(x-\beta)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)}$$

③ Integration by Substitution

$$\cos(x \cdot e^x) \quad x \cdot e^x \rightarrow t$$

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Q. $I = \int \frac{3^{\tan^{-1}x}}{1+x^2}\,dx$

$\Rightarrow I = \int 3^{\tan^{-1}x} \frac{1}{1+x^2}\cdot dx$

$= \int 3^t \cdot dt$

$= \frac{3^t}{\log_e 3} + C$

$I = \frac{3^{\tan^{-1}x}}{\log_e 3} + c$ //

$\tan^{-1}x = t$

differentiating w.r.t 'x'

$\frac{d}{dx}\tan^{-1}x = \frac{d}{dx}t$

$\frac{1}{1+x^2} = \frac{dt}{dx}$

$\frac{1}{1+x^2}\cdot dx = dt$ //

$\int a^x dx = \frac{a^x}{\log_e a} + c$

Q. $I = \int \frac{1}{9\cos^2 x + 4\sin^2 x} dx$

$$\Rightarrow I = \int \frac{\frac{1}{\cos^2 x}}{\frac{9\cos^2 x + 4\sin^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{\frac{9\cos^2 x}{\cos^2 x} + \frac{4\sin^2 x}{\cos^2 x}} dx$$

$\frac{1}{\cos x} = \sec x$

$\frac{\sin x}{\cos x} = \tan x$

$$= \int \frac{\sec^2 x}{9 + 4\tan^2 x} dx$$

$$= \int \frac{1}{9 + 4t^2} \cdot dt$$

$$= \int \frac{1}{4\left(\frac{9}{4} + t^2\right)} \cdot dt$$

$$= \frac{1}{4} \int \frac{1}{\frac{9}{4} + t^2} \cdot dt$$

$\tan x = t$

diff. w.r.t 'x'

$\frac{d}{dx} \tan x = \frac{d}{dx} t$

$\sec^2 x = \frac{dt}{dx}$

$\sec^2 x \cdot dx = dt$

$$= \frac{1}{4}\left[\frac{1}{3/2} \cdot \tan^{-1}\left(\frac{t}{3/2}\right)\right] + c$$

$\int \frac{1}{a^2 + x^2} \cdot dx = \frac{1}{a} \cdot \tan^{-1}\left(\frac{x}{a}\right) + c$

$a^2 = \frac{9}{4} \Rightarrow a = \frac{3}{2}$

$x^2 = t^2 \Rightarrow x = t$

$$I = \frac{1}{4}\left[\frac{1}{3/2} \cdot \tan^{-1}\left(\frac{\tan x}{3/2}\right)\right] + c \quad //$$

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Q. $I = \int x \cdot \tan^{-1}x \cdot dx$

$$\int u \cdot v \, dx = u \int v \cdot dx - \int \left[\frac{d}{dx}u \cdot \int v \cdot dx \right] dx$$

L
I = $\tan^{-1}x = u$
A = $x = v$
T
E

$$I = \int \tan^{-1}x \cdot x \cdot dx = \tan^{-1}x \int x^1 \cdot dx - \int \left[\frac{d}{dx}\tan^{-1}x \int x \cdot dx \right] \cdot dx$$

$$= \tan^{-1}x \cdot \left[\frac{x^{1+1}}{1+1} \right] - \int \left[\frac{1}{1+x^2} \cdot \left(\frac{x^{1+1}}{1+1} \right) \right] dx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$= \tan^{-1}x \cdot \frac{x^2}{2} - \int \left[\frac{1}{1+x^2} \cdot \frac{x^2}{2} \right] dx + C$$

$$I = \frac{x^2}{2} \cdot \tan^{-1}x - \frac{1}{2} \int \left[\frac{x^2}{1+x^2} \right] \cdot dx + C$$

$\downarrow$ I_1

$$I_1 = \int \frac{x^2}{1+x^2} \cdot dx$$

$$= \int \frac{(x^2+1)-1}{(x^2+1)} \cdot dx$$

$$= \int \left[\frac{(x^2+1)}{(x^2+1)} - \frac{1}{x^2+1} \right] dx$$

$$= \int \left[1 - \frac{1}{x^2+1} \right] dx$$

$$= \int 1 \cdot dx - \int \frac{1}{1+x^2} \cdot dx$$

$$I_1 = x - \tan^{-1}x + c$$

$$\therefore I = \frac{x^2}{2} \cdot \tan^{-1}x - \frac{1}{2} I_1$$

$$I = \frac{x^2}{2} \cdot \tan^{-1}x - \frac{1}{2} \times (x - \tan^{-1}x) + c$$

Q. $I = \int \frac{1}{1+\cos(2x)} \cdot dx$

$\Rightarrow$

$\tan x = t$

$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1-t^2}{1+t^2}$

$dx = \frac{1}{1+t^2} \cdot dt$

$\therefore I = \int \frac{1}{1+\frac{(1-t^2)}{(1+t^2)}} \cdot \frac{1}{(1+t^2)} \cdot dt$

$\therefore I = \int \frac{1}{\frac{1(1+t^2)+1(1-t^2)}{(1+t^2)}} \cdot \frac{1}{(1+t^2)} \cdot dt$

$\therefore I = \int \frac{1}{1(1+t^2)+1(1-t^2)} \cdot dt$

$= \int \frac{1}{1+t^2+1-t^2} \cdot dt$

$= \int \frac{1}{2} \cdot dt$

$= \frac{1}{2} \int 1 \cdot dt$

$= \frac{1}{2} t + c$

$I = \frac{1}{2} \cdot \tan x + c$ //

Q. $I = \int \cos(\log x) \cdot dx$

$I = \int 1 \cdot \cos(\log x) \cdot dx$

Hint to solve

L $= \cos(\log x) = u$
I
A $= 1 = v$
T
E

$$\int u \cdot v \, dx = u \int v \cdot dx - \int \left[\frac{d}{dx} u \cdot \int v \cdot dx \right] \cdot dx$$

$$= \cos(\log x) \cdot \int 1 \cdot dx - \int \left[\frac{d}{dx} \cos(\log x) \cdot \int 1 \cdot dx \right] dx$$